

SM3 11.4: Law of Cosines

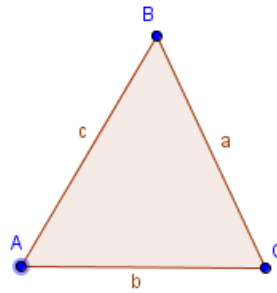
Given $\triangle ABC$, where the side opposite a given angle is named with the lowercase version of the letter name for the vertex,

the Law of Cosines states that

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$



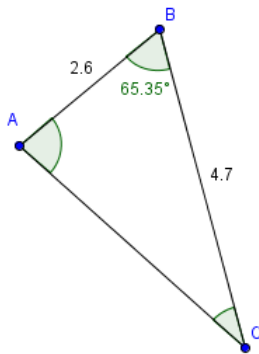
This is particularly useful for finding missing sides and angles in cases of a triangle where the pattern of known information follows a SAS (side, angle, side) or SSS (side, side, side) pattern.

Sometimes, a Law of Cosines problem will result in needing to do a Law of Sines calculation.

Problems: Find the missing measurements to the nearest hundredth using the Law of Cosines:

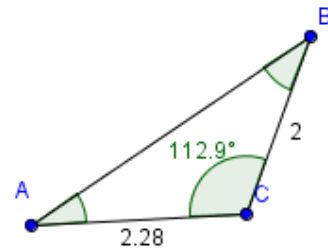
1)

$$\begin{aligned} m\angle A &= \\ m\angle C &= \\ b &= \end{aligned}$$



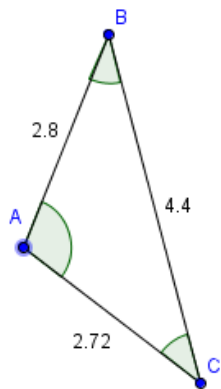
2)

$$\begin{aligned} m\angle A &= \\ m\angle B &= \\ c &= \end{aligned}$$



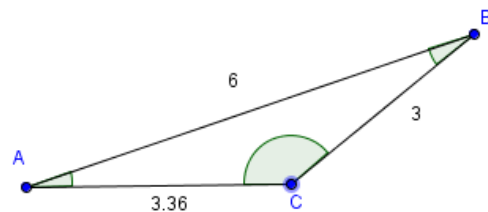
3)

$$\begin{aligned} m\angle A &= \\ m\angle B &= \\ m\angle C &= \end{aligned}$$



4)

$$\begin{aligned} m\angle A &= \\ m\angle B &= \\ m\angle C &= \end{aligned}$$



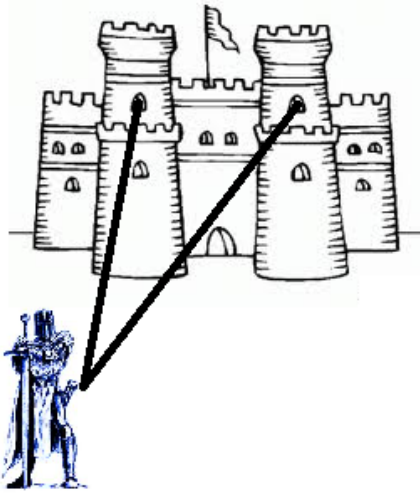
5) Harrison manufactures drums and drumsticks in several European cities. His company has manufacturing plants in London, Paris, and Amsterdam and he often visits to speak with his regional manager (and assistant to the regional manager) about optimizing their productivity to turn a higher profit. Harrison informs the pilot of his private jet that he needs to visit his London plant. They depart from Paris and travel 340 km to London. As the jet nears at London, Harrison reads that Amsterdam is having the worst banking crisis since 1763 so they turn 104° and fly another 358 km from London to Amsterdam.



Connect the points of travel to form a triangle and label the known measurements of the triangle. Use the Law of Cosines to approximate the distance from Paris to Amsterdam.

6) During a tennis doubles match, Lukas serves the ball a distance of 42 feet to one of his opponents, who is standing at the net. The opponent volleys the ball back over the net at a 48° angle directly to his brother, James. If James is 17 feet from the opponent, about how far is James from the position from which Lukas served the ball?

7) Nathan's betrothed, Emma, has been taken by an evil duke and locked in a tower in the duke's castle. Unfortunately, Nate has no idea which of the two upper towers Emma has been locked in. Nate figures that he could throw a rope tied to a hook up into each window and she could slide down. To keep from being spotted, Nate sets up somewhat to the side of the main portcullis. Being a great judge of distance, Nate approximates that the closer tower window is about 40 feet from his position while the farther window is about 60 feet from his position. If the castle windows are about 50 feet apart, what measure is the angle in between the two lines of rope?



8) If Abraham Lincoln, vampire hunter, thinks he had it rough... he should have seen what life was like for the original president: George Washington, dragon slayer! Legend is that back in George's day, he once shot down two flying dragons that were 100 feet apart with his cannon within moments of each other. That George could really reload quickly! If the first dragon was 150 feet from the cannon and the second dragon was 130 feet from the cannon, at what angle did George have to swivel his cannon from the first dragon to aim at the second dragon?

